**Math 72 8.5 Solving Nonlinear (Polynomial or Rational) Inequalities**

Objective

1) Solve polynomial inequalities. 8.5 - 1st
2) Solve rational inequalities. 8.5 - 2nd

**Goal:** Identify the intervals of values of x which make the inequality true, including or excluding the endpoints appropriately.

**Method**

Step 1: Is it a rational inequality? (Is there a denominator containing a variable)? If no, skip to Step 2.

- Set denominator equal to zero to find values of x which cause the expression to be undefined.
- These values are Critical Values.
- These values ALWAYS will be excluded from the solution (use parentheses in interval).

Step 2: Temporarily replace the inequality ($\leq, \geq, <, >$) with = and solve the Related Equation.

- If it's a rational inequality, multiply by the LCD to clear the fractions.
- Set equal to zero, factor, set factors = 0, isolate variable.
- These solutions are (also) called Critical Values.
- If the problem has $\leq, \geq$, these values will be included (brackets in interval).
- If the problem has $<, >$, these values will be excluded (parentheses on interval).

CAUTION: When two or more factors are multiplied, the resulting inequality is more complicated than if each factor is made into a linear inequality by itself. This is why we need a sign chart. [Do not write separate inequalities for each factor! That approach will receive no or little credit. ☹]

Step 3: Make a sign chart (number line).

- Plot all the critical values, in numerical order, using open circles if excluded and darkened circles if included.
- Notice the intervals (spaces) between critical values.
  - If there is one critical value, then there are two intervals.
  - If there are two critical values, then there are three intervals.
  - If there are three critical values, then there are four intervals. And so on…

Step 4: Test each interval.

- Option 1: Algebraic test.
  - Pick a test value of x in the interval (between critical values)
  - Substitute the test value into the inequality
  - If result is true, that interval is a solution interval.
  - If result is false, that interval is not a solution interval.
  - Mark the interval on the sign chart by shading (or √ or T) or unshaded (or X or F).
  - Continue to the next interval.

- Option 2: Graphing test.
  - If needed, rearrange the inequality so that there is a zero on the right side: $stuff \le 0$, or $stuff \ge 0$, or $stuff < 0$, or $stuff > 0$. [Caution: Zero on the left reverses the following logic!]
  - In the y= menu of the GC, graph the expression $y_1 = stuff$.
  - Check that the x-intercepts of your graph match the critical values found in Steps 1 and 2. (You may need to adjust the window.)
  - Between x-intercepts, the graph is either
    - above the x-axis, (positive y-coordinates are solutions $y_1 = stuff > 0$ or $y_1 = stuff \ge 0$)
    - below the x-axis, (negative y-coordinates are solutions of $y_1 = stuff < 0$ or $y_1 = stuff \le 0$)
  - Mark each interval on the sign chart.
    - If the graph is above the x-axis and
      - inequality is $> 0$ or $\ge 0$, the interval is a solution interval. (shade,√, T)
      - inequality is $< 0$ or $\le 0$, the interval is not a solution interval. (unshaded, X, F)
    - If the graph is below the x-axis and
      - inequality is $< 0$ or $\le 0$, the interval is a solution interval. (shade, √, T)
      - inequality is $> 0$ or $\ge 0$, the interval is not a solution interval. (unshaded, X, F)

- Option 3: Numerical test with GC.
  - If needed, rearrange the inequality so that there is a zero on the right side: $stuff \le 0$, or $stuff \ge 0$, or $stuff < 0$, or $stuff > 0$. [Caution: Zero on the left reverses the following logic!]
  - In the y= menu of the GC, graph the expression $y_1 = stuff$.
  - In TBLSET, set Independent Variable to ASK.
  - Pick a test value of x in the interval (between critical values)
  - Type test value into TABLE.
  - Mark interval on the sign chart.
    - If the y-value is positive and
      - inequality is $> 0$ or $\ge 0$, the interval is a solution interval. (shade,√, T)
      - inequality is $< 0$ or $\le 0$, the interval is not a solution interval. (unshaded, X, F)
    - If the y-value is negative and
      - inequality is $< 0$ or $\le 0$, the interval is a solution interval. (shade,√, T)
      - inequality is $> 0$ or $\ge 0$, the interval is not a solution interval. (unshaded, X, F)
  - Continue to the next interval.

<u>Step 5:</u> Write the solutions using interval or set notation, whichever the instructions request.

- When using interval notation,
  - Use [ or ] for included endpoints and ( or ) for excluded endpoints.
  - If there are more than one solution interval, use a union symbol: $\cup$, Ex: $[2,3] \cup [5,7]$
  - Use $\infty, -\infty$ if appropriate.
  - Do not use variables or inequalities $\le, \ge, <, >$
- When using set notation,
  - Use $\le$ for included endpoints and < for excluded endpoints
  - Use a comma before writing the next inequality, Ex: $\{x : 2 \le x \le 3, 5 \le x \le 7\}$
  - Use a variable (typically x).
  - Do not use $\infty, -\infty$.

**Math 70 8.4 Examples and Practice**

Solve the inequality. Write the solution in

a) Interval notation
b) Set notation

1) $x^2 - 3x - 10 > 0$

2) $\frac{x+2}{x-3} \leq 0$

3) $x^2 \geq 4x$

4) $\frac{x^2+6}{5x} \geq 1$

5) $4x^3 + 16x^2 < 9x + 36$

Math 70 8.4

① $x^2 - 3x - 10 > 0$

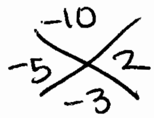
<u>step 1</u>: Is it rational? No — there is no fraction.

<u>step 2</u>: Solve related equation.

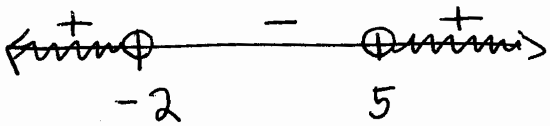
$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x = 5, \quad x = -2$$




<u>step 3</u>: sign chart



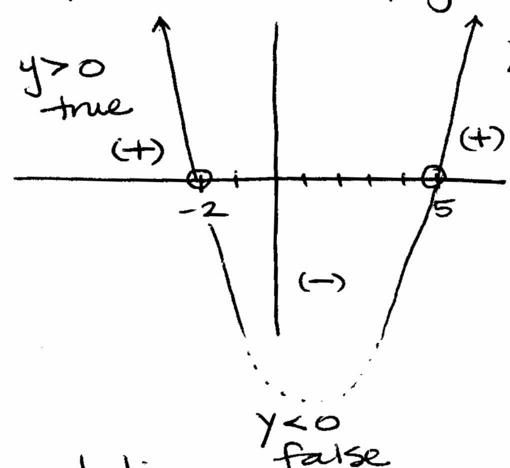

exclude with o because > (not ≥)

<u>step 4</u>: $y = x^2 - 3x - 10$ in GC

option 1, 3: test algebraically or numerically in GC

| x | y | | |
|---|---|---|---|
| -3 | 8 | + | true |
| 0 | -10 | − | false |
| 6 | 8 | + | true |

Option 2: test by graphing




<u>step 5</u>: Write solution

$(-\infty, -2) \cup (5, \infty)$ interval notation

$\{x : x < -2 \text{ or } x > 5\}$ set notation.

Math 70 8.4

② $\dfrac{x+2}{x-3} \le 0$

<u>step 1</u>: rational? yes

denom = 0

$x-3=0$

$x=3$. exclude b/c divide by zero

<u>step 2</u>: related equation

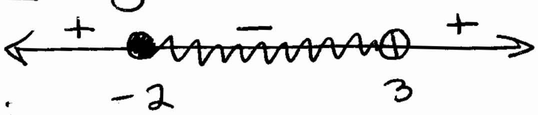
$\dfrac{x+2}{x-3} = 0$ cross-mult or LCD-mult

$x+2 = 0(x-3)$

$x+2=0$

$x=-2$ include b/c $\le$

<u>step 3</u>: sign chart

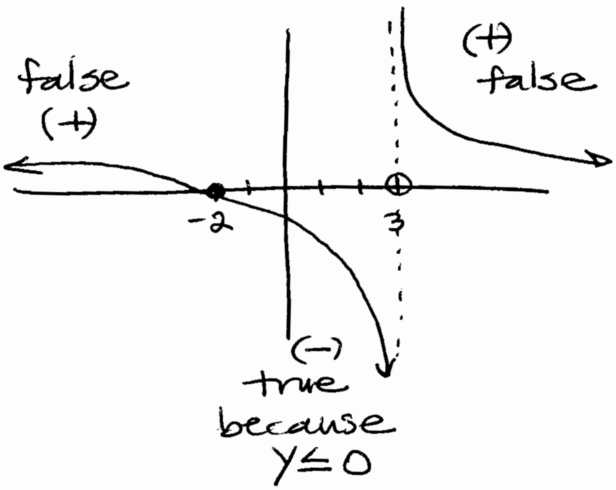



<u>step 4</u>:

<u>option 1,3</u>: test algebraically or numerically

| x | y |
|---|---|
| −3 | 1/6 (+) false |
| 0 | −2/3 (−) true |
| 4 | 6 (+) false |

<u>option 2</u>: test graphically




<u>step 5</u>:

$\boxed{[-2,3)}$ interval notation

$\boxed{\{x : -2 \le x < 3\}}$ set notation

Math 70 8.4

③ $x^2 \geq 4x$

Step 1: rational? no.

Step 2: related equation.

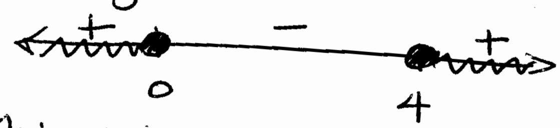
$x^2 = 4x$ — see $x^2$ = quadratic

$x^2 - 4x = 0$ — set = 0

$x(x-4) = 0$ — GCF $x$

$x = 0 \quad x = 4$

Step 3 sign chart




● include because ≥

Step 4: set = 0 before graphing!

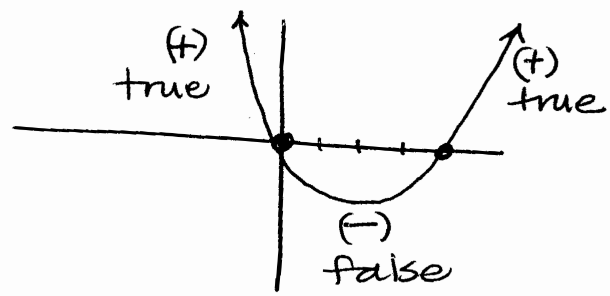
$x^2 - 4x \geq 0$

$y = x^2 - 4x$ in GC

option 1

| x | y | |
|---|---|---|
| −1 | 5 | + true |
| 2 | −4 | − false |
| 5 | 5 | + true |

option 2




Step 5:

$(-\infty, 0] \cup [4, \infty)$ interval notation

$\{x : x \leq 0 \text{ or } x \geq 4\}$ set notation

Math 70 8.4

④ $\frac{x^2+6}{5x} \geq 1$

Step 1: rational? yes

denom = 0

$5x = 0$

$x = 0$ exclude b/c divide by zero.

Step 2: solve related equation.

$\frac{x^2+6}{5x} = 1$ cross-mult or mult by LCD

$x^2+6 = 1 \cdot 5x$

$x^2+6 = 5x$ see $x^2 \rightarrow$ quadratic

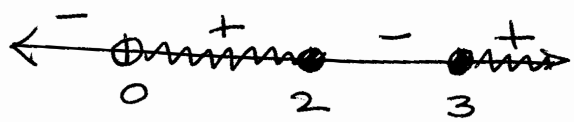
$x^2-5x+6=0$ set = 0.

$(x-2)(x-3)=0$ 6, −2, −3, −5

$x=2 \quad x=3$

include because ≥

Step 3




Step 4: set ≥ 0 before graphing/using table

use original inequality!

$\frac{x^2+6}{5x} \geq 1$

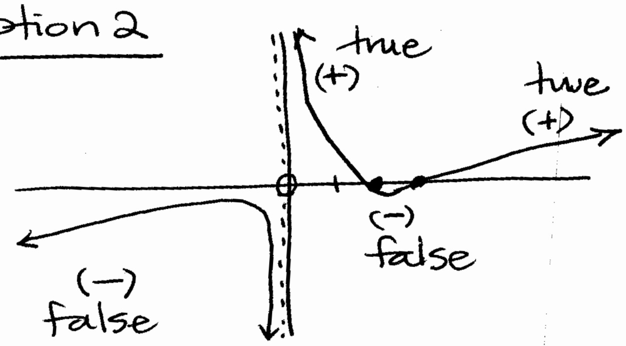
$\frac{x^2+6}{5x} - 1 \geq 0$

$y = (x^2+6)/(5x) - 1$

option 1

| x | y | |
|---|---|---|
| −1 | −2.4 | (−) false |
| 1 | 4 | (+) true |
| 2.5 | −.02 | (−) false |
| 4 | .1 | (+) true |

option 2




Step 5: $(0,2] \cup [3,\infty)$ interval notation

$\{x: 0 < x \leq 2 \text{ or } x \geq 3\}$ set notation

Math 70 8.4

(5) $4x^3 + 16x^2 < 9x + 36$

step 1: rational? no.

Step 2: related equation

$4x^3 + 16x^2 = 9x + 36$ — see $x^3$

$4x^3 + 16x^2 - 9x - 36 = 0$ — set $= 0$

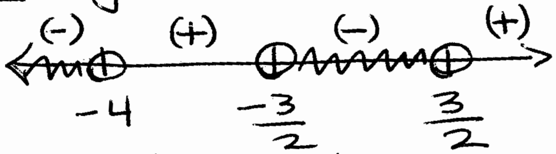
$4x^2(x+4) - 9(x+4) = 0$ — 4 terms, factor by grouping

$(x+4)(4x^2 - 9) = 0$ — difference of squares

$(x+4)(2x-3)(2x+3) = 0$

$x = -4, \frac{3}{2}, \frac{-3}{2}$ — exclude because <

step 3: sign chart.




set original = 0 before GC

$4x^3 + 16x^2 - 9x - 36 < 0$

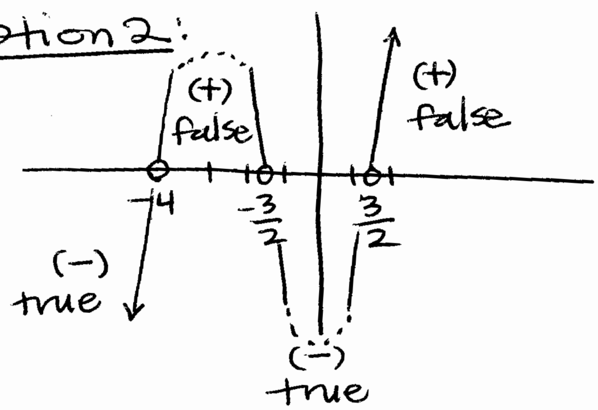
$Y = 4x^3 + 16x^2 - 9x - 36$ in GC

Step 4

option 1

| X | Y | |
|---|---|---|
| −5 | −91 | (−) true |
| −2 | 14 | (+) false |
| 0 | −36 | (−) true |
| 2 | 42 | (+) false |

option 2:




step 5:

$(-\infty, -4) \cup \left(-\frac{3}{2}, \frac{3}{2}\right)$ interval notation

$\left\{x : x < -4 \text{ or } -\frac{3}{2} < x < \frac{3}{2}\right\}$ set notation.

Rev 8-8-13

Name ______________________

Date ______________________

**TI-84+ GC 27 Testing Polynomial Inequalities in One Variable**

Objectives: Review algebraic method for solving polynomial inequalities
Review the signs of y-coordinates of points in each quadrant
Relate the solution of the polynomial inequality to the graph of the polynomial equation
Use the graph of the polynomial equation to determine solutions of the inequality

A polynomial inequality in one variable has expressions which have natural number exponents and one of the following symbols: $<, >, \le, \ge$ . For example: $5x^3 - 3x^2 + 5x < 7$ or $4y^2 - y + 1 \ge 0$.

To solve a polynomial inequality, find all the values of the variable which, when substituted, make the inequality true. Since there are usually an infinite number of values, we use interval notation.

Algebraic method:
Step 1: Rewrite so one side of the inequality is zero by adding or subtracting terms to both sides.
Step 2: Solve the related equation (change the inequality to an equal sign) for the **critical values**, by factoring or quadratic formula if polynomial is a quadratic.
Step 3: Draw a number line and plot the critical values. Use open circles ∘ if the inequality is < or > (exclude the critical values), and closed circles • if the inequality is ≤ or ≥ (include critical values).
Step 4: Test the intervals between the critical values by substituting a test point. If true, the interval is part of the solution set. If false, interval is not part of the solution set. Shade the intervals of the number line that correspond to solutions.
Step 5: Write the interval(s) for the solution set, using [ or ] for • (inclusion) and ( or ) for ∘ (exclusion), connected by union symbols ∪ if needed.

Step 4 of the algebraic method can often be done more quickly on the GC. Let's explore how.

1) If a point (x,y) is in Quadrant I, is the y-coordinate positive or negative?

2) If a point (x,y) is in Quadrant II, is the y-coordinate positive or negative?

3) If a point (x,y) is in Quadrant III, is the y-coordinate positive or negative?

4) If a point (x,y) is in Quadrant IV, is the y-coordinate positive or negative?

5) If a point (x,y) is on the x-axis, what is the y-coordinate?

6) If $y > 0$, in which quadrants might the point (x,y) lie?

7) If $y < 0$, in which quadrants might the point (x,y) lie?

8) If the y-coordinate of a point (x,y) is positive, is the point above or below the x-axis?

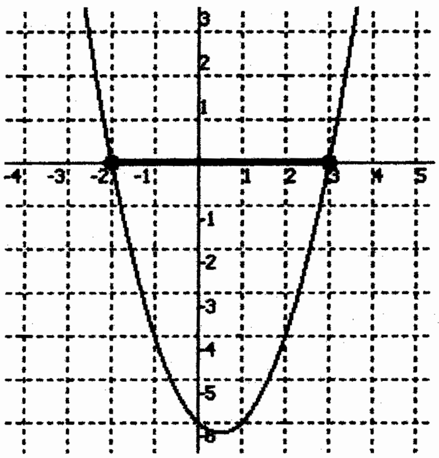
9) If the y-coordinate of a point (x,y) is negative, is the point above or below the x-axis?

10) If the y-coordinate of a point (x,y) is zero, where is the point?

## TI-84+ GC 27 Testing Polynomial Inequalities in One Variable p.2

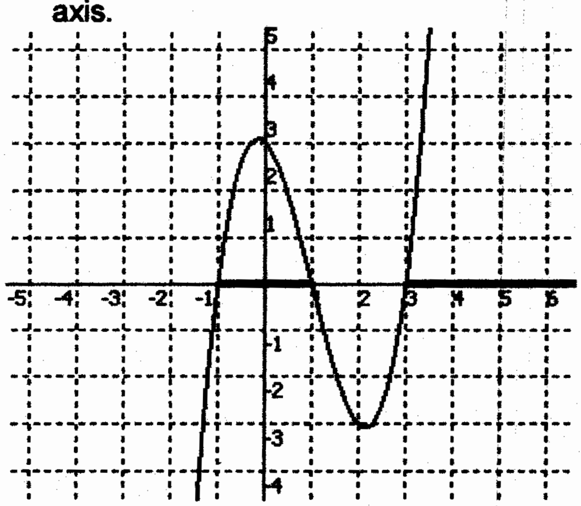
Recall: Set y=0 in $y=x^2-5x-6$ then solve for x to find the x-intercepts.
The solutions of $x^2-5x-6=0$ are the x-coordinates of the x-intercepts.

11) On this graph, circle the parts of the graph which have negative y-coordinates, $y \le 0$. Notice that the x-coordinates for the circled points have been shaded, on the x-axis.



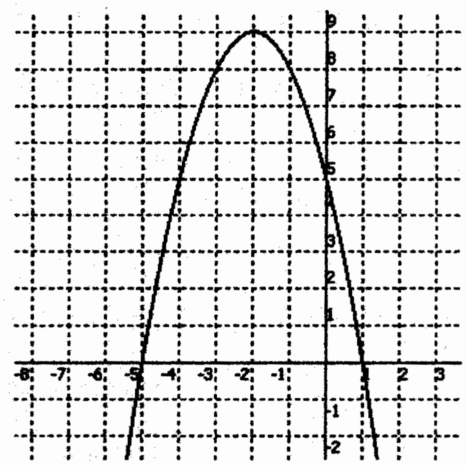
12) Write the interval notation for the shaded values of x in the previous question. Include the endpoints.

13) On this graph, circle the parts of the graph which have positive y-coordinates, $y \ge 0$. Notice that the x-coordinates for the circled points have been shaded, on the x-axis.

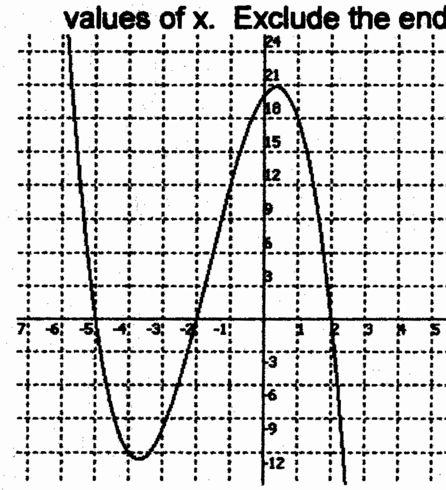


14) Write the interval notation for the shaded values of x in the previous question. Include the endpoints. (Hint: It's two intervals with a union.)

15) On this graph, shade the x-axis for all values of x where $y < 0$ on the graph. Then write the interval notation for these values of x. Exclude the endpoints.



16) On this graph, shade the x-axis for all values of x where $y > 0$ on the graph. Then write the interval notation for these values of x. Exclude the endpoints.



# TI-84+ GC 27 Testing Polynomial Inequalities in One Variable p.3

Solutions of $\underbrace{x^2-5x-6}_{y}>0$ correspond to points (x,y) on the graph of $y=x^2-5x-6$ where $y>0$.

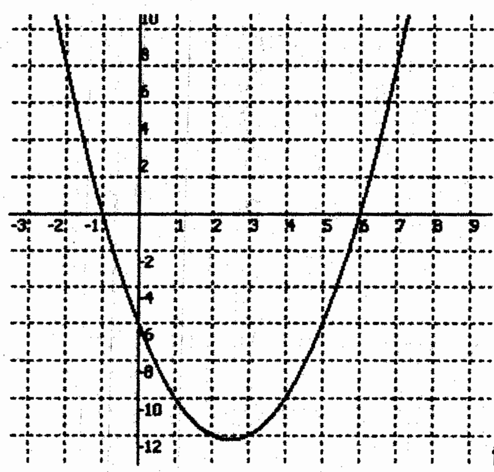
Solutions of $\underbrace{x^2-5x-6}_{y}<0$ correspond to points (x,y) on the graph of $y=x^2-5x-6$ where $y<0$.

These are called strict inequalities. Solutions of strict inequalities do not include the critical values. Interval notation uses parentheses ( ).

Solutions of $\underbrace{x^2-5x-6}_{y}\le 0$ correspond to points where $y<0$ or $y=0$.

Solutions of $\underbrace{x^2-5x-6}_{y}\ge 0$ correspond to points where $y>0$ or $y=0$.

These are called non-strict inequalities. Solutions of non-strict inequalities include the critical values, because they make the expression equal to zero. Interval notation uses brackets [ ].




Use this graph of $y=x^2-5x-6$ to write the solutions to the following inequalities in interval notation:

17) $x^2-5x-6>0$

18) $x^2-5x-6<0$

19) $x^2-5x-6\le 0$

20) $x^2-5x-6\ge 0$

| Summary | Strict inequalities<br>exclude endpoints<br>parentheses ( ) and ∘ | Non-Strict inequalities<br>include endpoints<br>use brackets [ ] and • |
|---|---|---|
| $y>0$ or $y\ge 0$<br>Quadrants I and II | $y>0$<br>Quadrants I and II<br>use parentheses ( ) | $y\ge 0$<br>Quadrants I and II<br>use brackets [ ] |
| $y<0$ or $y\le 0$<br>Quadrants III and IV | $y<0$<br>Quadrants III and IV<br>use parentheses ( ) | $y\le 0$<br>Quadrants III and IV<br>use brackets [ ] |

<u>Solving Polynomial Inequalities using Algebra and Graphing</u> (hybrid method)
Step 1: Rewrite so one side of the inequality is zero by adding or subtracting terms to both sides.
Step 2: Solve the related equation (change the inequality to an equal sign) for the **critical values**, by factoring or quadratic formula if polynomial is a quadratic.
Step 3: Draw a number line and plot the critical values. Use open circles ∘ if the inequality is < or > (exclude the critical values), and closed circles • if the inequality is ≤ or ≥ (include the critical values).
Step 4: Graph the function. Check that x-intercepts match your critical values. Shade the intervals of the number line that correspond to solutions.
Step 5: Write the interval(s) for the solution set, using [ or ] for • and ( or ) for ∘, connected by union symbols ∪ if needed.

21) Solve $x^2 - 4x \geq 12$ using the hybrid method.

a) Make one side of the inequality zero by adding or subtracting.

b) Factor to find the critical values.

c) Draw a number line and plot the critical values. Do you plot with ∘ or •?

d) Graph the function $y = x^2 - 4x - 12$ on your GC. Are your answers from b) the x-intercepts? If not, check your algebra and check your Y= menu.

e) Are the solutions for this problem in QI and QII or in QIII and QIV? Shade the solutions on the number line in c).

f) Write intervals for the shaded parts. Do you use parentheses ( ) or brackets [ ]?

22) Solve $x^3 - 9x^2 + 11x + 21 < 0$ using the hybrid method. If you can't factor, find the critical values from the x-intercepts on the graph.

Rev 8-8-13

Name ______________________________

Date ______________________________

**TI-84+ GC 28 Testing Rational Inequalities in One Variable**

Objectives: Review algebraic method for solving rational inequalities
Use correct notation for critical values from the numerator
Use correct notation for critical values from the denominator
Use the graph of the polynomial equation to determine solutions of the inequality

A rational inequality in one variable has expressions which are fractions with variables in one or more denominators and one of the following: $<, >, \le, \ge$ . For example: $\frac{2x+3}{x-5} - \frac{-x+7}{x+5} < 7$ or $\frac{2}{x} \ge 0$.

To solve a rational inequality, find all the values of the variable which, when substituted, make the inequality true. Since there are usually an infinite number of values, we use interval notation.

Algebraic method:
Step 1: Rewrite so one side of the inequality is zero by adding or subtracting terms to both sides. Add or subtract the fractions using a common denominator so that one simplified fraction remains.

Step 2: Solve for the critical values.
Step 2a: Solve the related equation (change the inequality to an equal sign).
Short cut: Set the numerator equal to zero and solve. (x-intercepts)
Step 2b: Find the values that are not in the domain of the rational expression.
Short cut: Set the denominator equal to zero and solve.

Step 3: Draw a number line and plot the critical values.
Step 3a: For the critical values which come from the numerator, use open circles ∘ if the inequality is < or > (exclude the critical values), and closed circles • if the inequality is ≤ or ≥ (include critical values).
Step 3b: For the critical values which come from the denominator, always use open circles ∘

Step 4: Test the intervals between the critical values by substituting a test point. If true, the interval is part of the solution set. If false, interval is not part of the solution set. Shade the intervals of the number line that correspond to solutions.

Step 5: Write the interval(s) for the solution set, using [ or ] for • and ( or ) for ∘, connected by union symbols ∪ if needed.

Step 4 can often be done more quickly on the GC, just as for polynomial inequalities.

1) If $y > 0$, in which quadrants might the point (x,y) lie?

2) If $y < 0$, in which quadrants might the point (x,y) lie?

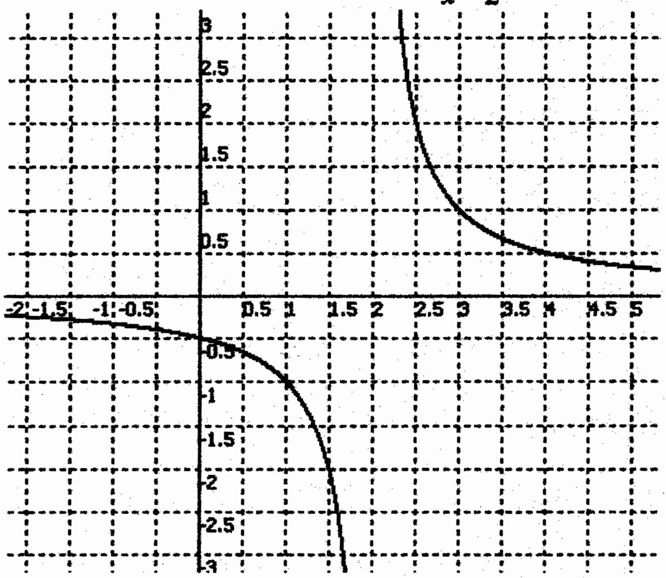
3) If the y-coordinate of a point (x,y) is positive, is the point above or below the x-axis?

4) If the y-coordinate of a point (x,y) is negative, is the point above or below the x-axis?

5) If the y-coordinate of a point (x,y) is zero, where is the point?

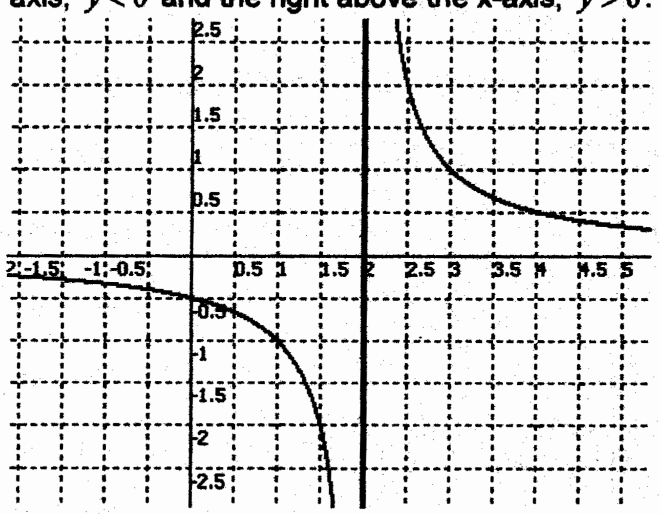
6) If the y-coordinate is undefined, where is the point?

Let's look at the graph of $f(x)=\dfrac{1}{x-2}$, as if we were solving $\dfrac{1}{x-2}\le 0$.

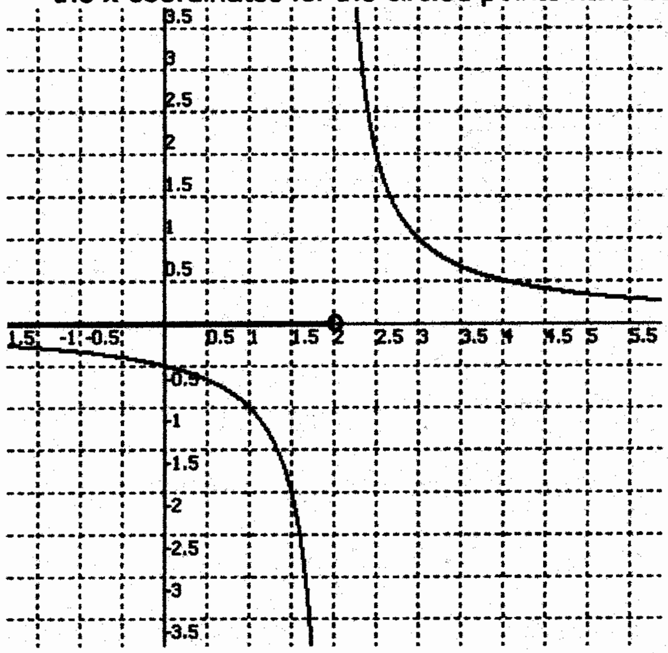


Find the critical values:
Step 2a: Set the numerator equal to zero: 1=0 is a contradiction, so we have no critical values from the numerator. (There are no x-intercepts.)
Step 2b: Set the denominator equal to zero: $x-2=0$ gives x=2 as a critical value.

Let's add the line x=2 to the graph. Notice that the graph of $f(x)=\dfrac{1}{x-2}$ goes off the graph at x=2. This is called a vertical asymptote. It divides the graph into its two parts: the left part is below the x-axis, $y<0$ and the right above the x-axis, $y>0$.



7) On this graph, circle the parts of the graph which have negative y-coordinates, $y \le 0$. Notice that the x-coordinates for the circled points have been shaded on the x-axis.




8) Write the interval notation for the shaded values of x in the previous question. Exclude the endpoint, which is the critical value at x=2.

In the next several questions, we will solve the rational inequality $\frac{x-1}{x^2-x-6} \ge 0$.

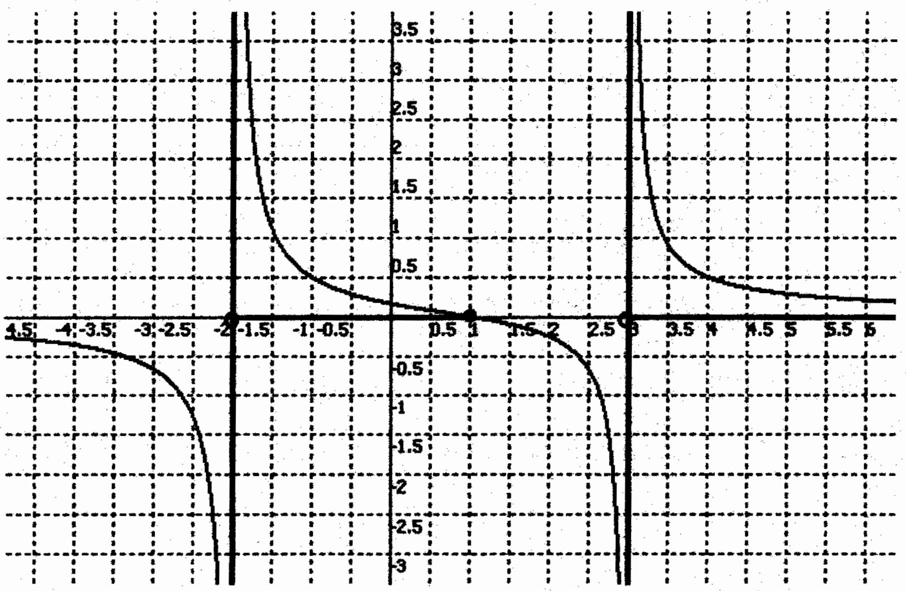
9) Find the critical value(s) from the numerator. Will these be closed circles and brackets, or open circles and parentheses?

10) Find the critical value(s) from the denominator. Will these be closed circles and brackets, or open circles and parentheses?

11) Graph $y_1 = \frac{x-1}{x^2-x-6}$ on your GC and compare to the information you just found. Does your graph have x-intercept(s) at your critical values from numerator? Does your graph have vertical asymptotes at your critical values from the denominator? (Hint: If your graph doesn't match your work, check that you used correct parentheses in your Y= menu.)

12) The graph of $y_1 = \frac{x-1}{x^2-x-6}$ is given below, with the vertical asymptotes added. Compare this to the graph on your GC.

On this graph, circle the parts of the graph which have positive y-coordinates, $y \geq 0$. Notice that the x-coordinates for the circled part have been shaded on the x-axis. Write the interval notation for the shaded values of x, using correct notation for the endpoints.




Solutions of $\frac{x-1}{x^2-x-6} > 0$ correspond to points (x,y) on the graph of $y = \frac{x-1}{x^2-x-6}$ where $y > 0$.
Solutions of $\frac{x-1}{x^2-x-6} < 0$ correspond to points (x,y) on the graph of $y = \frac{x-1}{x^2-x-6}$ where $y < 0$.
These are called <u>strict inequalities</u>. Solutions of strict inequalities do not include any of the critical values. Interval notation uses parentheses ( ) for all endpoints.

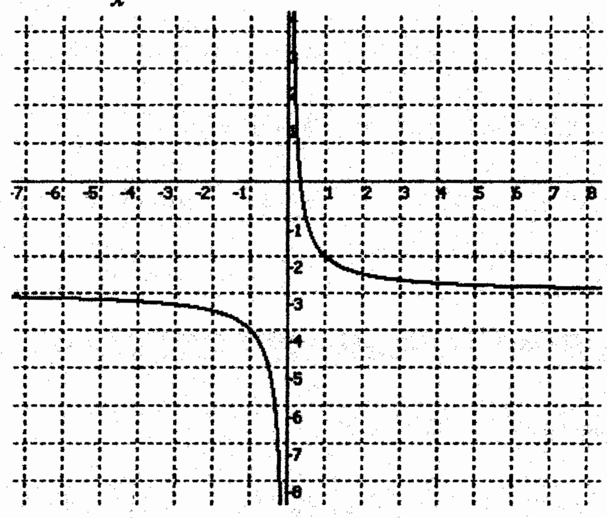
Solutions of $\frac{x-1}{x^2-x-6} \leq 0$ correspond to points where $y < 0$ or $y = 0$.
Solutions of $\frac{x-1}{x^2-x-6} \geq 0$ correspond to points where $y > 0$ or $y = 0$.
These are called <u>non-strict inequalities</u>.

Solutions of non-strict inequalities include the critical values from the numerator, because they make the expression equal to zero. Interval notation uses brackets [ ] for the critical values from the numerator.

Solutions of non-strict inequalities do NOT include the critical values from the denominator, because these make the expression undefined. (Undefined is not greater or less than 0, so the inequality is never true at these points.)
<u>Always</u> use parentheses ( ) for the interval notation of critical values from the denominator.

| **Summary** | Strict inequalities: critical values from numerator exclude endpoints parentheses ( ) and ∘ | Non-Strict inequalities: critical values from numerator include endpoints use brackets [ ] and • | Critical values from denominator: exclude endpoints parentheses ( ) and ∘ |
|---|---|---|---|
| $y > 0$ or $y \geq 0$ Quadrants I and II | $y > 0$ Quadrants I and II use parentheses ( ) | $y \geq 0$ Quadrants I and II use brackets [ ] | Quadrants I and II use parentheses ( ) |
| $y < 0$ or $y \leq 0$ Quadrants III and IV | $y < 0$ Quadrants III and IV use parentheses ( ) | $y \leq 0$ Quadrants III and IV use brackets [ ] | Quadrants III and IV use parentheses ( ) |

13) Find the critical values for $\frac{1-3x}{x} > 0$ algebraically. Use your answers for the next questions.




Use this graph of $y = \frac{1-3x}{x}$ and your critical values to write the solutions to the following inequalities using interval notation.

14) $\frac{1-3x}{x} > 0$

15) $\frac{1-3x}{x} \geq 0$

16) $\frac{1-3x}{x} < 0$

17) $\frac{1-3x}{x} \leq 0$

# TI-84+ GC 28 Testing Rational Inequalities in One Variable p.6

<u>Solving Rational Inequalities using Algebra and Graphing</u> (hybrid method)

Step 1: Same as algebraic method

Step 2: Solve for the critical values.
Step 2a: Either same as algebraic method, or use GC to find x-intercepts, using Zero.
Step 2b: same as algebraic method

Step 3: Same as algebraic method

Step 4: Graph the function. Check that x-intercepts and vertical asymptotes match your critical values. Shade the intervals of the number line that correspond to solutions.

Step 5: Same as algebraic method

18) Solve $\frac{x+2}{x-3}+\frac{x-1}{x+3}\geq\frac{-2x^2+x+18}{x^2-9}$ using the hybrid method.

a) Make one side of the inequality zero by adding or subtracting.

b) Write each fraction with the common denominator. FOIL and simplify each numerator.

c) Add or subtract the fractions to get one fraction with the common denominator.

d) Find the critical value(s) from the numerator.

e) Find the critical values(s) from the denominator.

f) Draw a number line and plot the critical values. Be careful with ∘ or • !

Continued next page

g) Graph the function $y=\frac{4x^2-9}{x^2-9}$ on your GC. Are your answer(s) from d) the x-intercept(s)? Are your answer(s) from e) the vertical asymptote(s)? If not, check your algebra and check your Y= menu.

h) Are the solutions for this problem in QI and QII or in QIII and QIV? Shade the solutions on the number line in c).

i) Write intervals for the shaded parts. Be careful with parentheses ( ) and brackets [ ].

19) Solve $\frac{x+1}{x-4}+\frac{x-2}{x+4}<\frac{-2x^2+x+32}{x^2-16}$ using the hybrid method.

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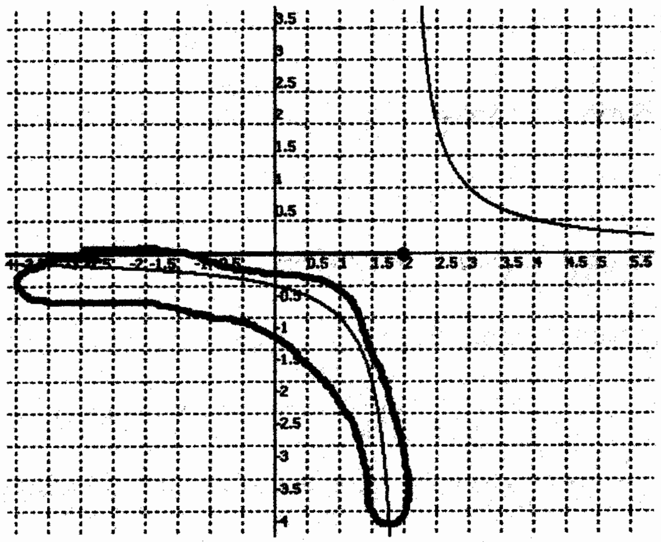
# TI-84+ GC 28 Testing Rational Inequalities in One Variable p.8 Solutions

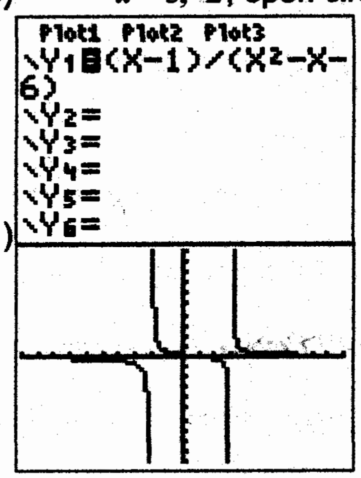
1) QI or QII
2) QIII or QIV
3) above the x-axis
4) below the x-axis
5) on the x-axis
6) it's undefined, so it's not on the graph.

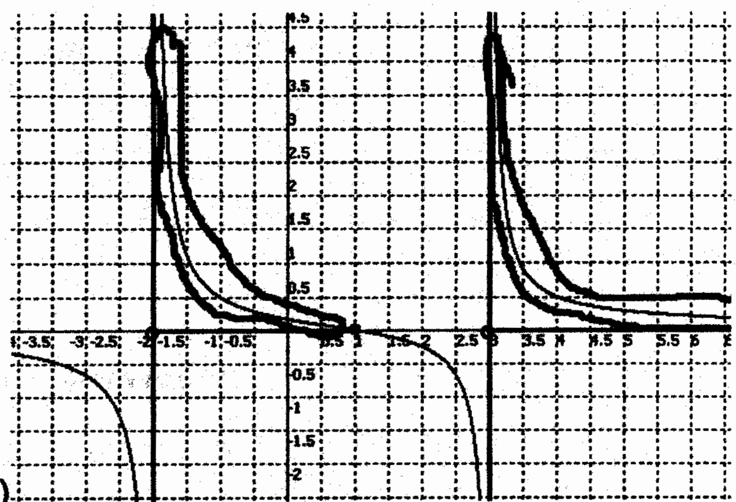
7) 

8) $(-\infty,2)$

9) x=1, closed circles and brackets

10) $x=3,-2$, open circles and parentheses

11) 


12) 

$(-2,1]\cup(3,\infty)$

13) $x=0,\frac{1}{3}$

14) $\left(0,\frac{1}{3}\right)$

15) $\left(0,\frac{1}{3}\right]$

16) $(-\infty,0)\cup\left(\frac{1}{3},\infty\right)$

17) $(-\infty,0)\cup\left[\frac{1}{3},\infty\right)$

18)

a. $\frac{x+2}{x-3}+\frac{x-1}{x+3}-\frac{-2x^2+x+18}{x^2-9}\geq 0$

b. $\left(\frac{x+2}{x-3}\right)\cdot\left(\frac{x+3}{x+3}\right)+\left(\frac{x-1}{x+3}\right)\left(\frac{x-3}{x-3}\right)-\frac{-2x^2+x+18}{x^2-9}\geq$

$\frac{x^2+5x+6}{x^2-9}+\frac{x^2-4x+3}{x^2-9}-\frac{-2x^2+x+18}{x^2-9}\geq 0$

c. $\frac{4x^2-9}{x^2-9}\geq 0$

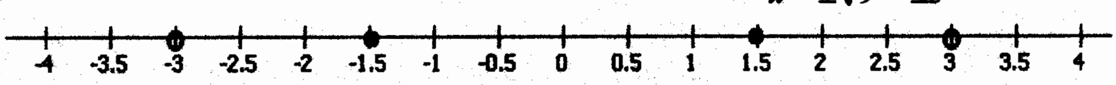
d. $4x^2-9=0$

$(2x-3)(2x+3)=0$

$x=\pm\frac{3}{2}$

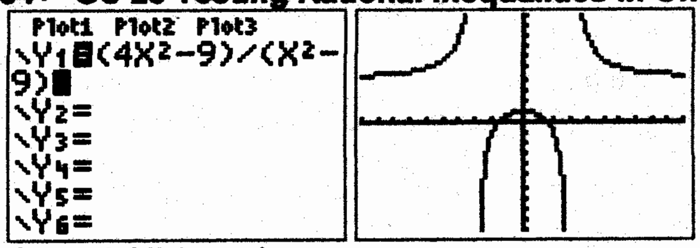
e. $x^2-9=0$

$x=\pm\sqrt{9}=\pm 3$

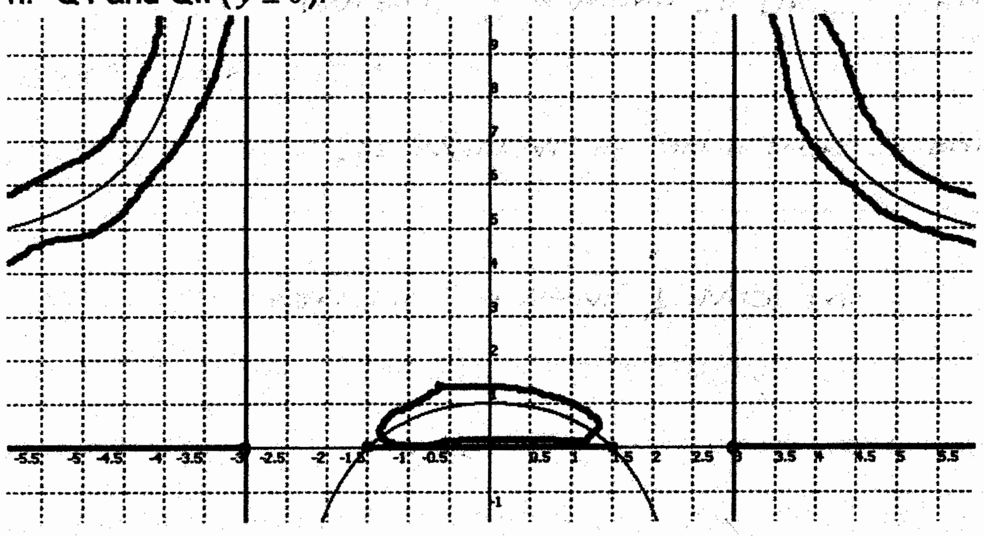
f. 

## TI-84+ GC 28 Testing Rational Inequalities in One Variable p.9 Solutions

g.




h. Q1 and QII ($y \geq 0$).




i. $(-\infty,-3)\cup\left[-\frac{3}{2},\frac{3}{2}\right]\cup(3,\infty)$

19) $y=\dfrac{4x^2-2x-20}{x^2-16}$, QIII and QIV ($y<0$), solution: $(-4,-2)\cup\left(\frac{5}{2},4\right)$